

Process: any activity or group of activities that takes one or more inputs, transforms them, and provides one or more outputs for its customers.

Operation: a group of resources performing all or part of one or more processes.

Operations management: the systematic design, direction, and control of processes that transform inputs into services and products for internal, as well as external, customers.

Types of customers / suppliers:

- Internal
- External

The service / product spectrum:

Differ Across Nature of Output and Degree of Customer Contact



- Physical, durable output
- Output can be inventoried
- Low customer contact
- Long response time
- Capital intensive
- Quality easily measured

- Intangible, perishable output
- Output cannot be inventoried
- High customer contact
- Short response time
- Labor intensive
- Quality not easily measured

Types of processes:

1. Core: a set of activities that delivers value to external customers.
 - a. Supplier relationship process
 - b. New service / product development process
 - c. Order fulfillment process
 - d. Customer relationship management
2. Support: a set of activities that provides inputs to the core processes.

Operations strategy overview:

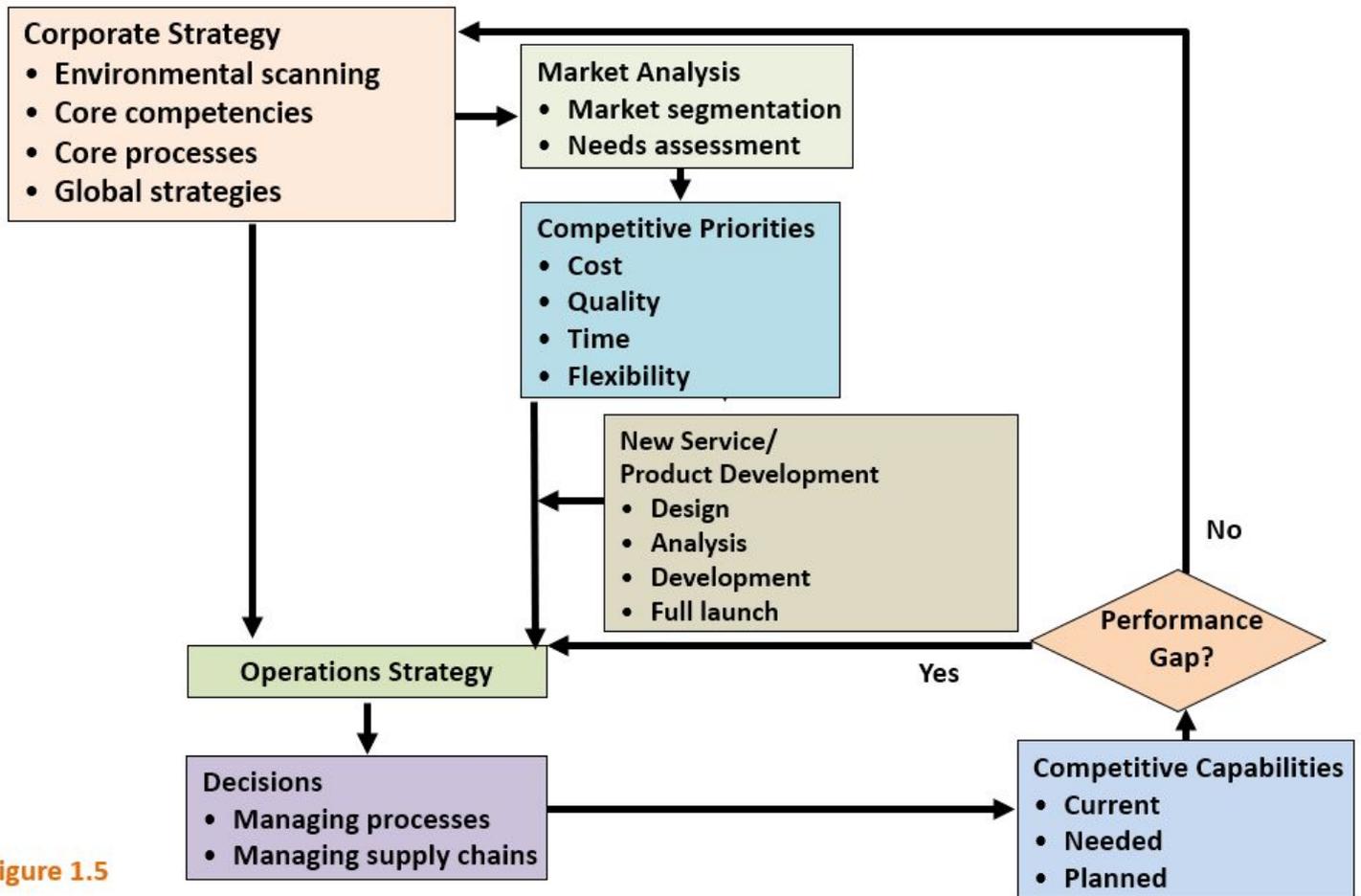


Figure 1.5

Developing core competencies:

1. Workforce (important for service organizations)
2. Facilities (important for manufacturing organizations)
3. Market and financial know how
4. Systems and technology (important for data intensive organizations)

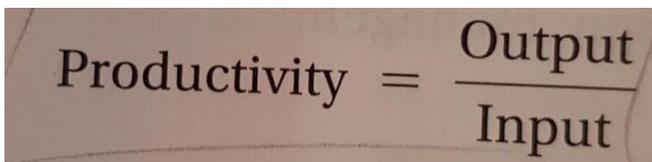
Competitive priorities:

1. Cost
 - a. Low cost operations
2. Quality
 - a. Top quality
 - b. Consistent quality
3. Time
 - a. Delivery speed
 - b. On-time delivery
 - c. Development speed
4. Flexibility
 - a. Customization
 - b. Variety
 - c. Volume flexibility

Order winner: a criterion customers use to differentiate the services or products of one firm from those of another.

Order qualifier: minimal level required from a set of criteria for a firm to do business in a particular market segment.

Productivity: the value of output produced divided by the values of input resources.

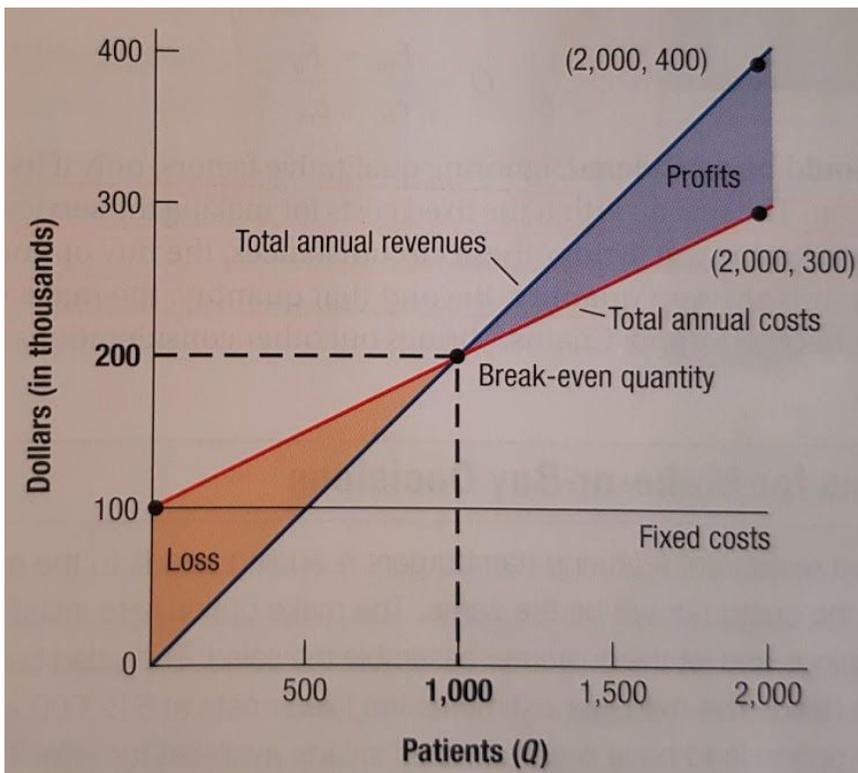

$$\text{Productivity} = \frac{\text{Output}}{\text{Input}}$$

CHAPTER 1A: DECISION MAKING MODELS

Break even analysis: helps the manager identify how much change in volume or demand before a second alternative becomes better than the first alternative.

Break even analysis:

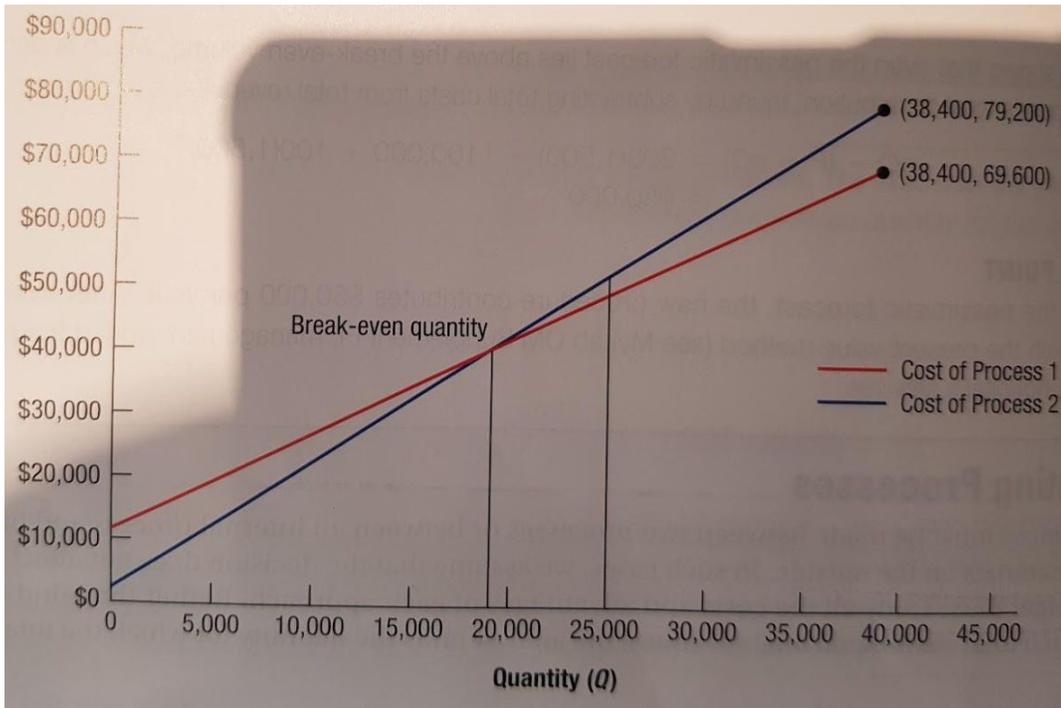
$$pQ = F + cQ$$
$$(p - c)Q = F$$
$$Q = \frac{F}{p - c}$$



Sensitivity (make or buy) analysis:

$$F_b + c_b Q = F_m + c_m Q$$

$$Q = \frac{F_m - F_b}{c_b - c_m}$$



Performance matrix: helps a manager deal with multiple criteria that cannot be evaluated with a single measure of merit, such as total cost or profit.

Performance Criterion	Weight (A)	Score (B)	Weighted Score (A × B)
Market potential	30	8	240
Unit profit margin	20	10	200
Operations compatibility	20	6	120
Competitive advantage	15	10	150
Investment requirement	10	2	20
Project risk	5	4	20
			Weighted score = 750

Decision theory: helps the manager choose the best alternative when outcomes are uncertain.

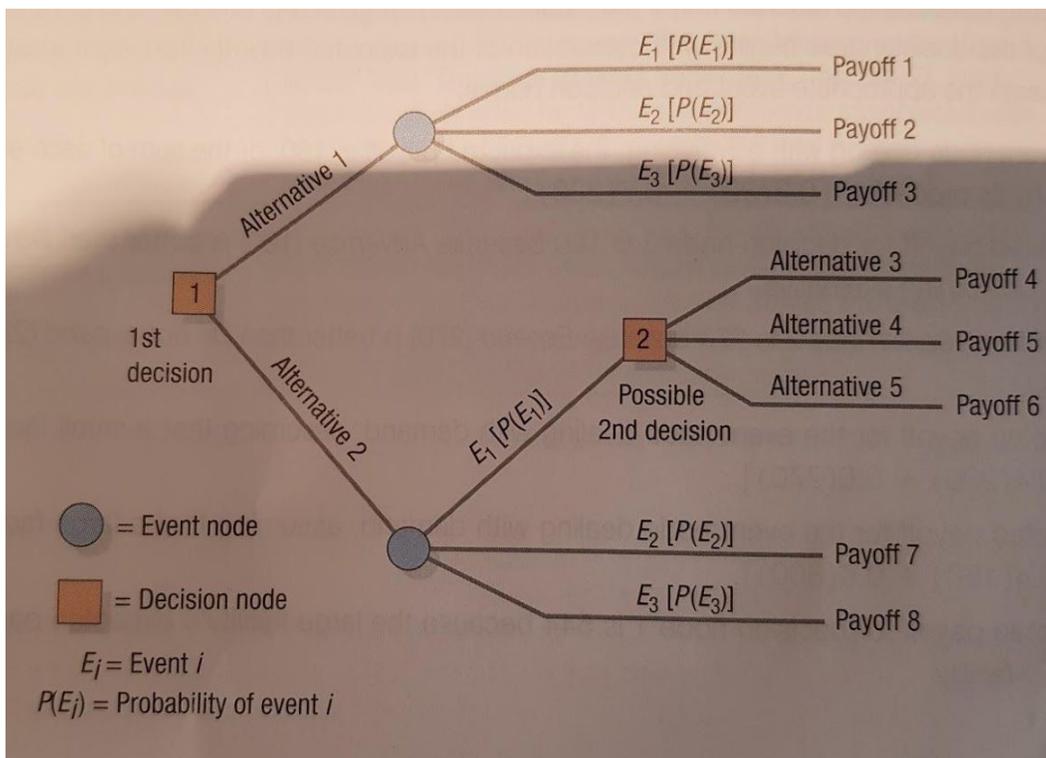
General process:

1. List the feasible alternatives
2. List the events
3. Calculate the payoff for each alternative in each event
4. Estimate the likelihood of each event
5. Select a decision rule

Situations:

1. Certainty: pick the alternative with the best payoff for the known event.
2. Uncertainty: unknown probabilities.
 - a. Maximin
 - b. Maximax
 - c. Laplace
 - d. Minimax regret
3. Risk: same as Laplace, but with the probabilities known.

Decision tree: helps the manager when decisions are made sequentially (e.g., when today's best decision depends on tomorrow's decisions and events).



CHAPTER 2: PROCESS STRATEGY AND ANALYSIS

Process strategy: the pattern of decisions made in managing processes so that they will achieve their competitive priorities.

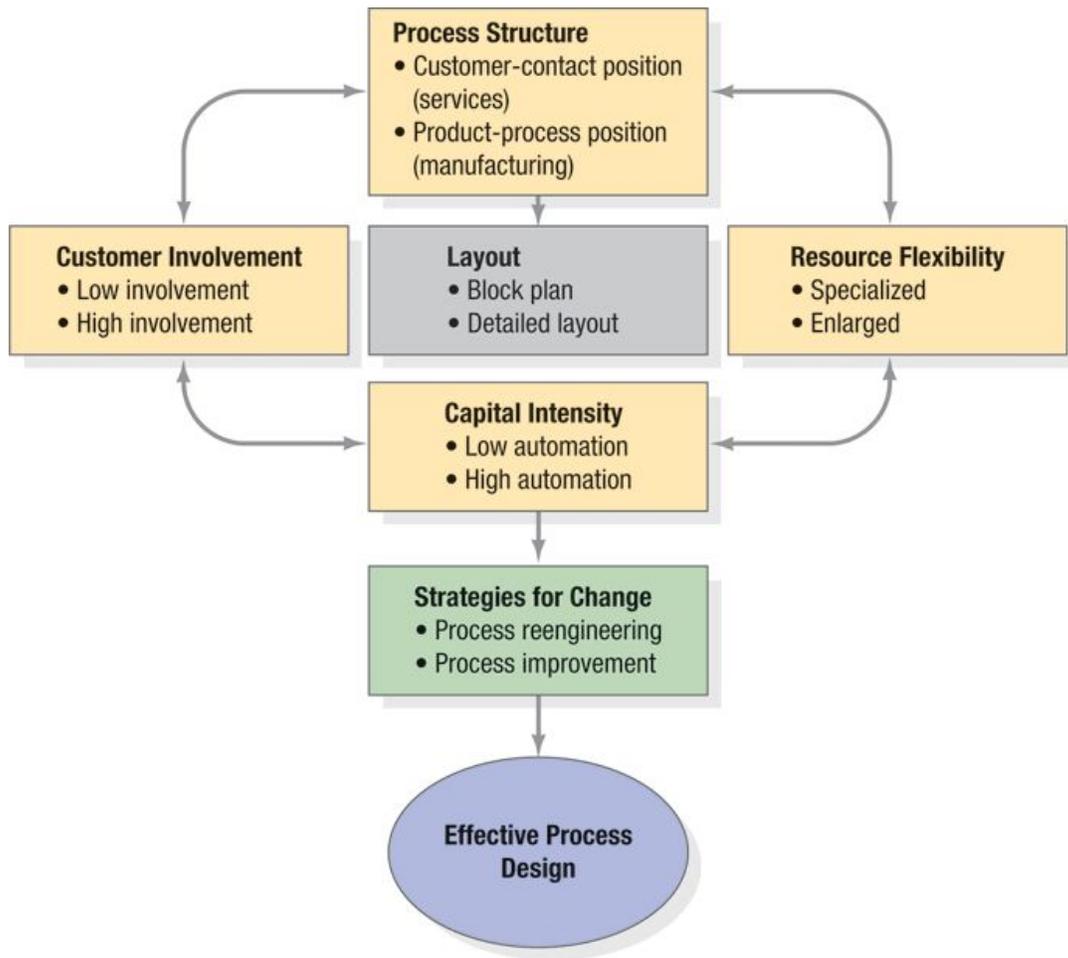
Process analysis: the documentation and detailed understanding of how work is performed and how it can be redesigned.

Process decisions:

1. Process structure
2. Customer involvement
3. Resource flexibility
4. Capital intensity

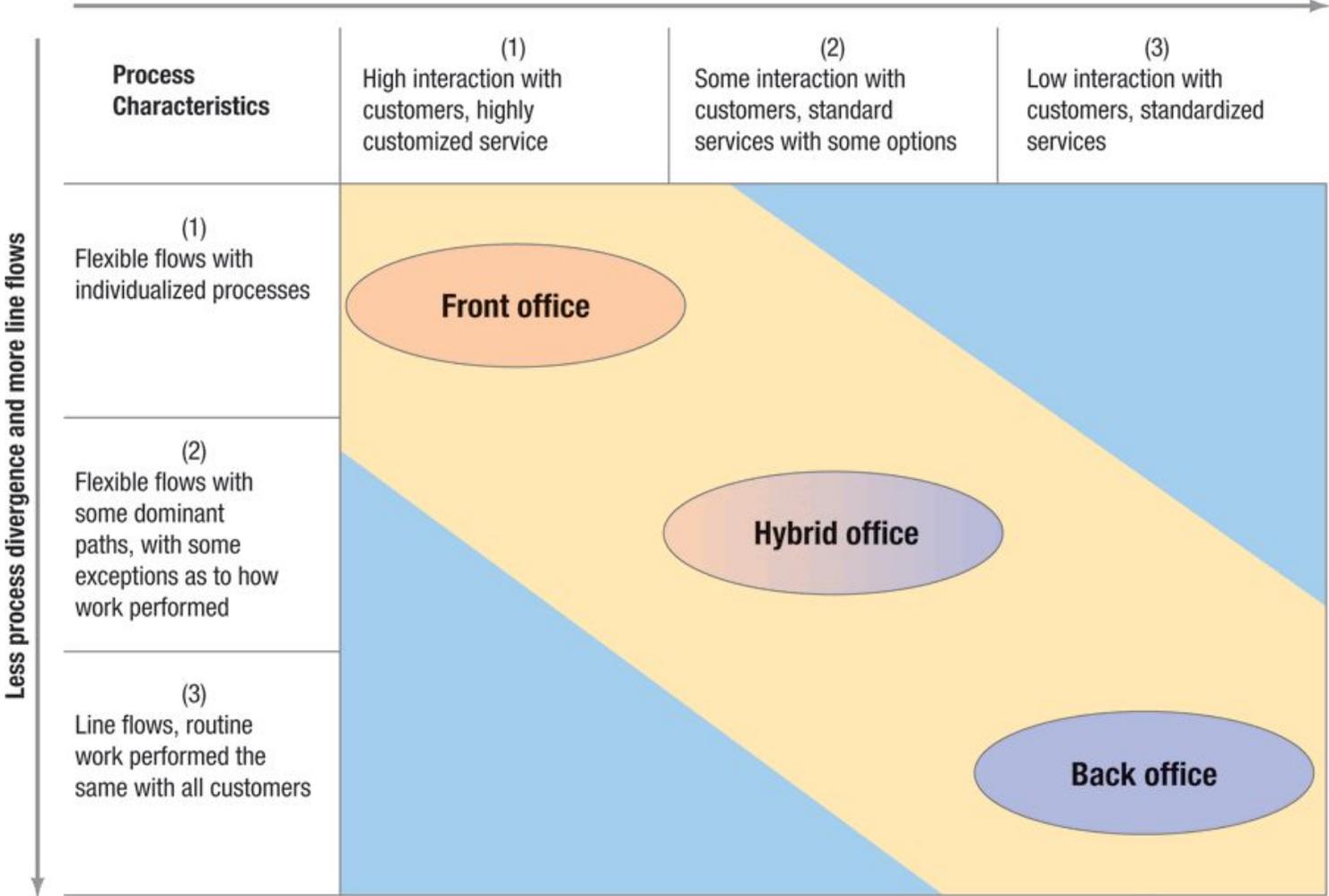
Strategies for analyzing and modifying processes:

1. Process improvement
2. Process reengineering

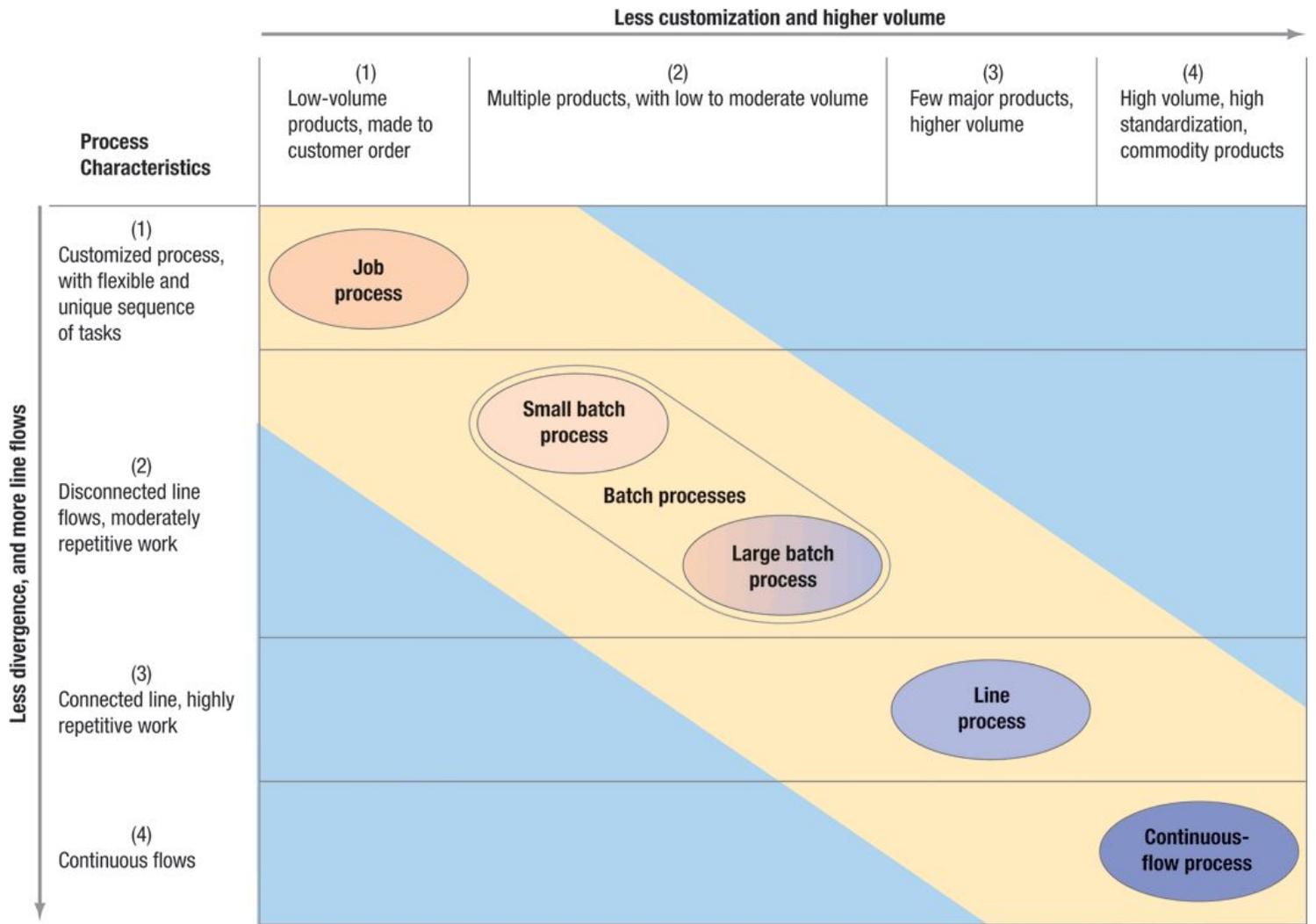


Customer-contact matrix:

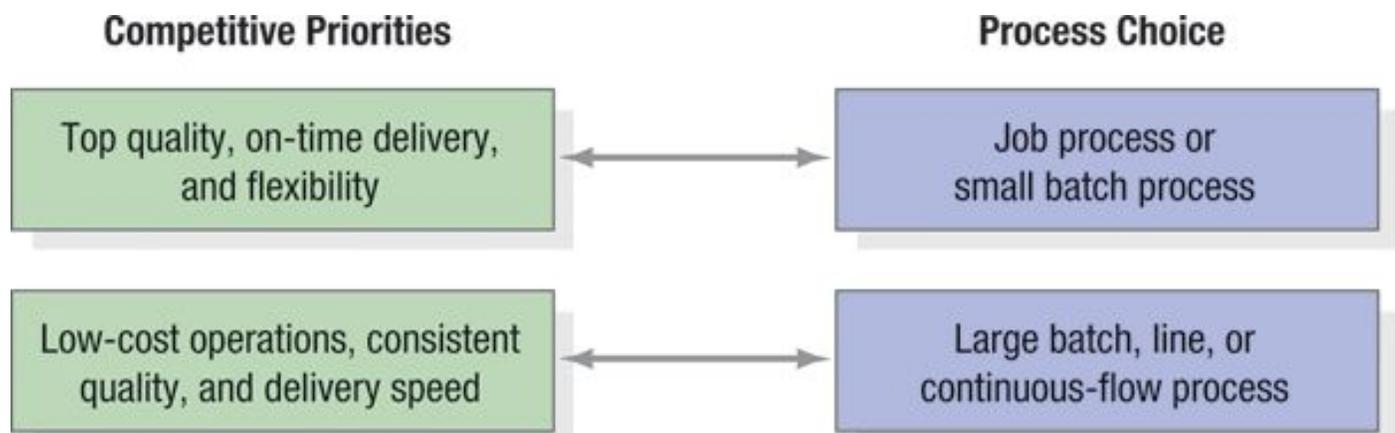
Less customer contact and customization



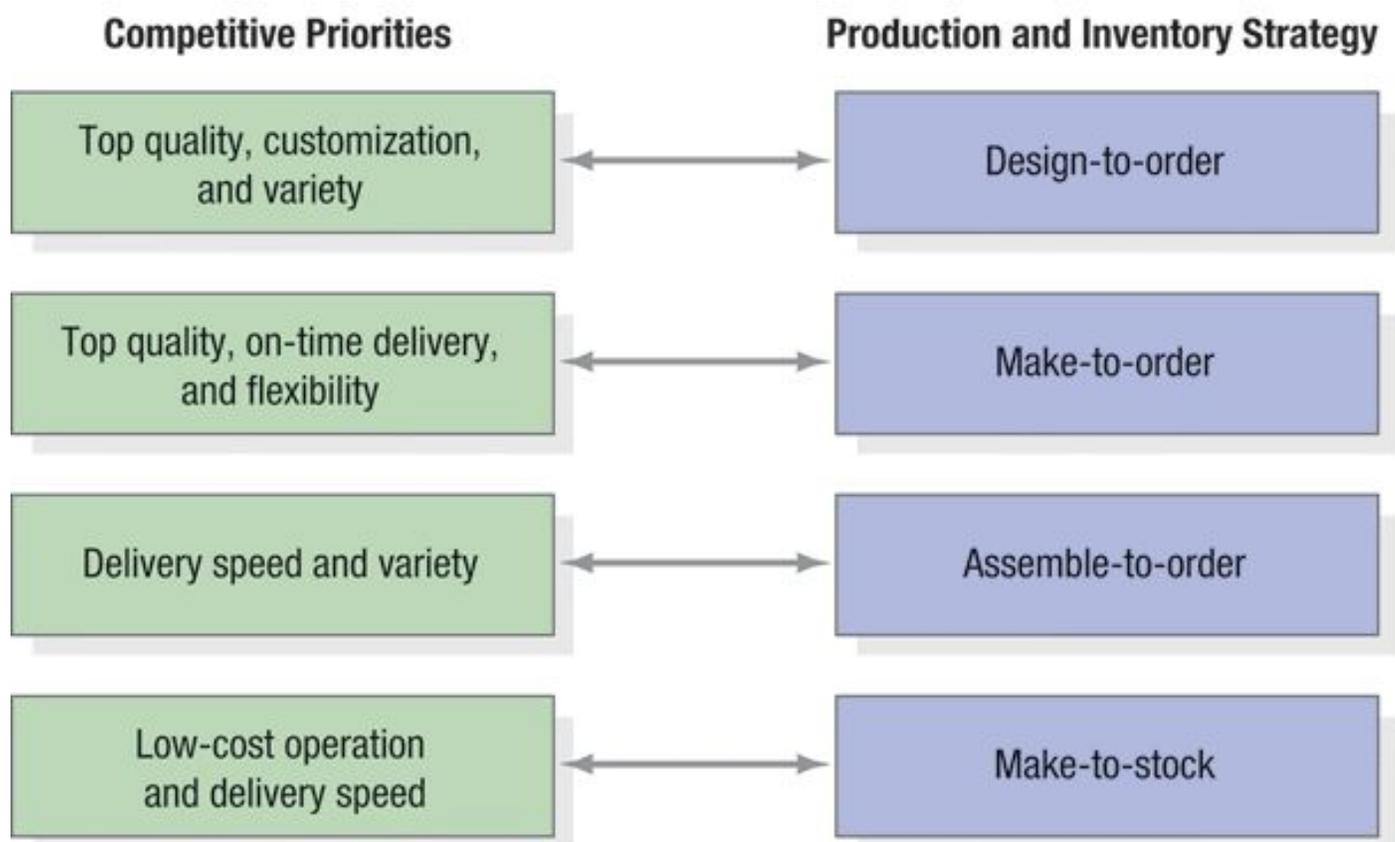
Product-process matrix:



Decision patterns for manufacturing processes:



(a) Links with Process Choice



(b) Links with Production and Inventory Strategy

Six Sigma:

- Disciplined, quantitative approach for continuous improvement and for establishing defined metrics in operations.
- A Six Sigma performance level means 3.4 defects per 1 million opportunities.
- The number 3.4 defects per 1 million is obtained by assuming that the specification limits are 6 standard deviations away from the target.
- Six Sigma problem solving phases:
 - Define
 - Measure
 - Analyze
 - Improve
 - Control

Learning curve: a line that displays the relationship between processing time and the cumulative quantity of a product or service produced.

$$k_n = k_1 n^b$$

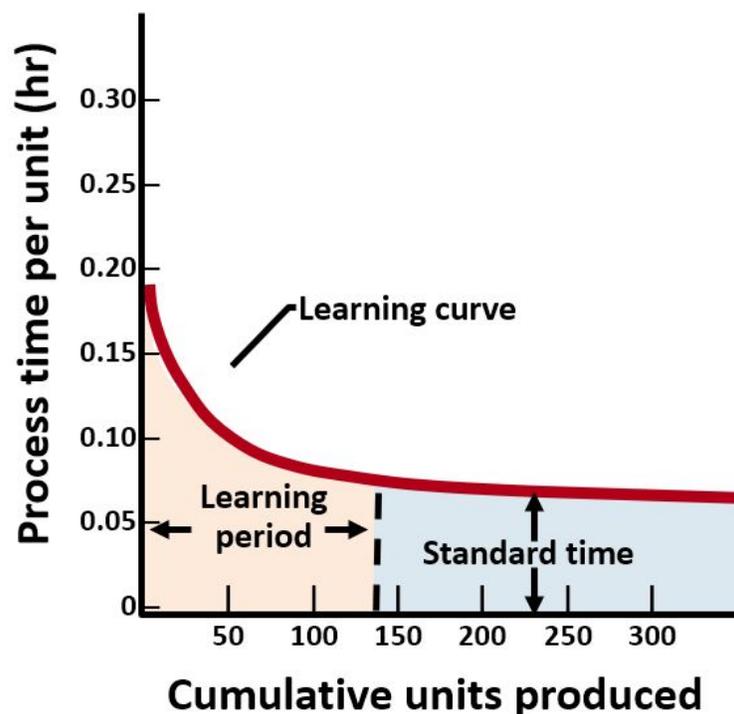
where

k_1 = direct labor hours for the first unit

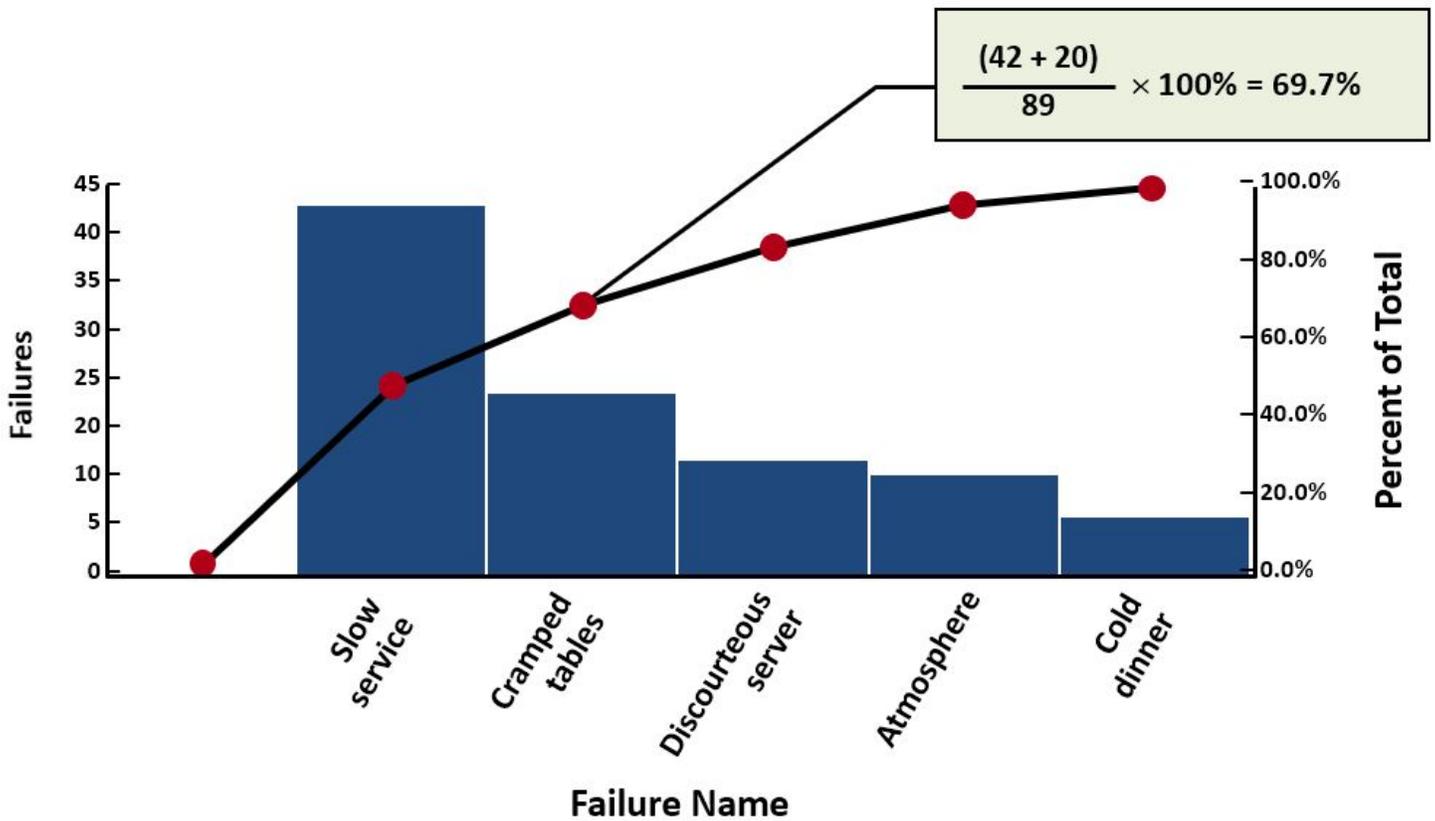
n = cumulative numbers of units produced

$$b = \frac{\log r}{\log 2}$$

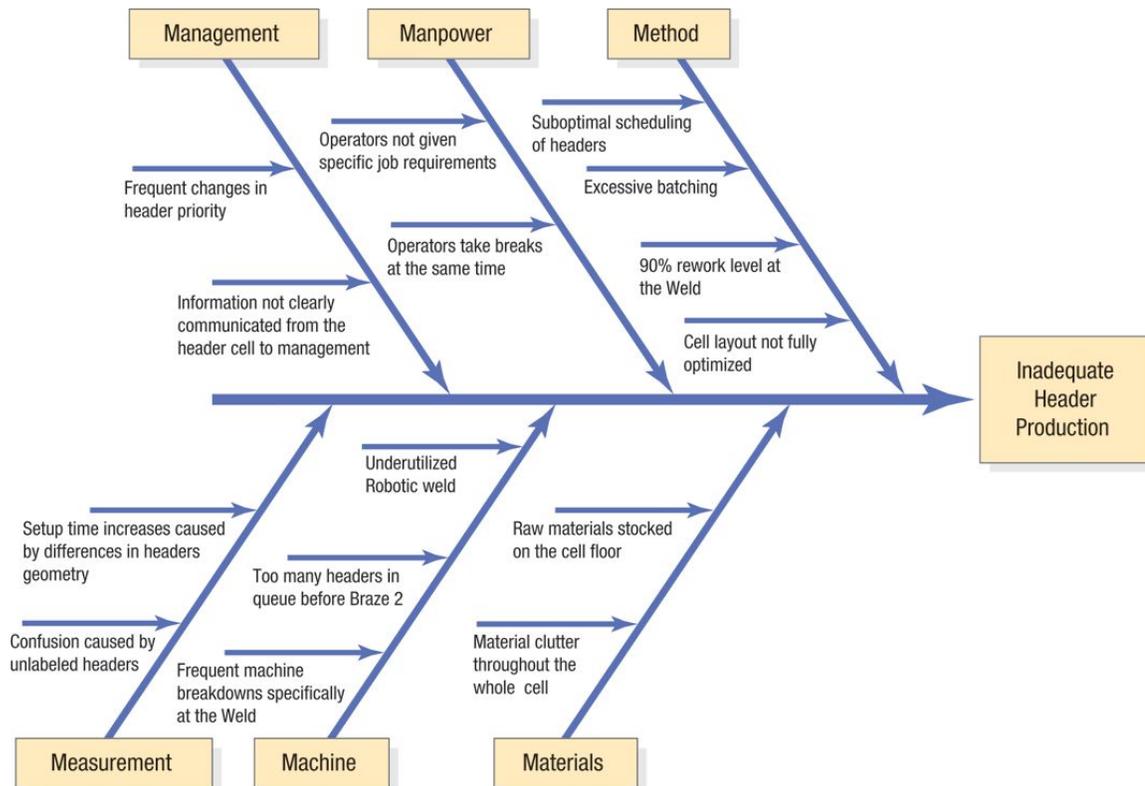
r = learning rate (as decimal)



Pareto chart:



Cause and effect diagram (i.e., fishbone diagram):



CHAPTER 3: MANAGING QUALITY

Types of costs:

- Prevention costs
- Appraisal costs
- Internal failure costs
- External failure costs
- Ethical failure costs

Total quality management – main principles:

1. Customer satisfaction
 - a. Conformance to specification
 - b. Value
 - c. Fitness for use
 - d. Support
 - e. Psychological impressions
2. Employee involvement
 - a. Cultural change
 - b. Teams
3. Continuous improvement
 - a. Plan
 - b. Do
 - c. study
 - d. Act

Acceptance sampling: the application of statistical techniques to determine whether a quantity of material should be accepted or rejected based on the inspection or test of a sample.

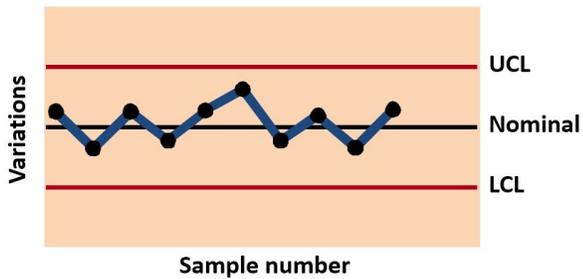
Acceptable quality level: the quality level desired by the consumer.

Statistical process control: the application of statistical techniques to determine whether a process is delivering what the customer wants.

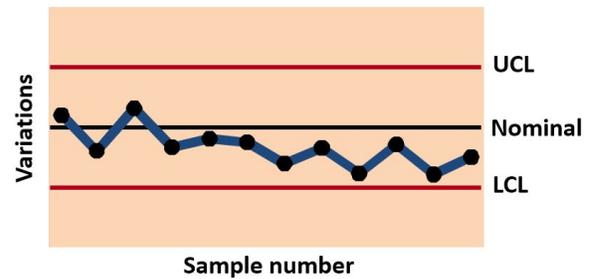
Types of variation causes:

1. Common causes
2. Assignable causes

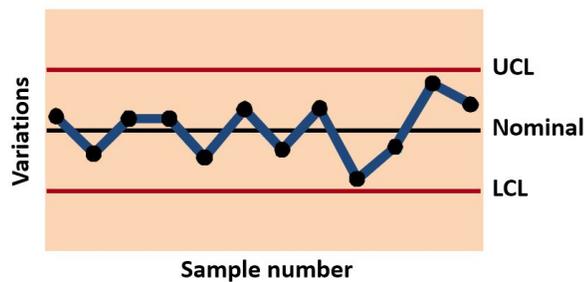
Control chart: a time-oriented diagram that is used to determine whether observed variations are abnormal.



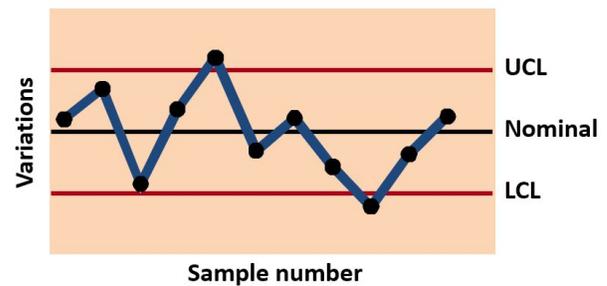
(a) Normal – No action



(b) Run – Take action



(c) Sudden change – Monitor



(d) Exceeds control limits – Take action

Type I and type II errors:

Table of error types		Null hypothesis (H_0) is	
		True	False
Decision About Null Hypothesis (H_0)	Accept	Correct inference (True Positive)	Type II error (False Negative)
	Reject	Type I error (False Positive)	Correct inference (True Negative)

Sample mean:

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

Sample standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}}$$

R-Chart: concerned with process variability.

$$\mathbf{UCL}_R = D_4\bar{R} \quad \text{and} \quad \mathbf{LCL}_R = D_3\bar{R}$$

where

\bar{R} = average of several past R values and the central line of the control chart

D_3, D_4 = constants that provide three standard deviation (three-sigma) limits for the given sample size

Size of Sample (n)	Factor for UCL and LCL for \bar{x} -Chart (A_2)	Factor for LCL for R -Chart (D_3)	Factor for UCL for R -Chart (D_4)
2	1.880	0	3.267
3	1.023	0	2.575
4	0.729	0	2.282
5	0.577	0	2.115
6	0.483	0	2.004
7	0.419	0.076	1.924
8	0.373	0.136	1.864
9	0.337	0.184	1.816
10	0.308	0.223	1.777

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X bar chart: concerned with process output.

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + A_2\bar{R} \quad \text{and} \quad \text{LCL}_{\bar{x}} = \bar{\bar{x}} - A_2\bar{R}$$

where

$\bar{\bar{x}}$ = central line of the chart, which can be either the average of past sample means or a target value set for the process

A_2 = constant to provide three-sigma limits for the sample mean

$$\text{UCL}_{\bar{x}} = \bar{\bar{x}} + z\sigma_{\bar{x}} \quad \text{and} \quad \text{LCL}_{\bar{x}} = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

where

$$\sigma_{\bar{x}} = \sigma/\sqrt{n}$$

σ = standard deviation of the process distribution

n = sample size

$\bar{\bar{x}}$ = central line of the chart

z = normal deviate number

P-Chart: concerned with proportions.

$$\sigma_p = \sqrt{\bar{p}(1 - \bar{p})/n}$$

\bar{p} = the center line on the chart

$$\text{UCL}_p = \bar{p} + z\sigma_p \quad \text{and} \quad \text{LCL}_p = \bar{p} - z\sigma_p$$

Process capability: the ability of the process to meet the design specifications for a service or a product.

Process Capability Index: An index that measures the potential for a process to generate defective outputs relative to either upper or lower specifications.

$$C_{pk} = \text{Minimum of } \left(\frac{\bar{x} - \text{Lower specification}}{3\sigma}, \frac{\text{Upper specification} - \bar{x}}{3\sigma} \right)$$

where

σ = standard deviation of the process distribution

The process capability index must be compared to a critical value to judge whether a process is capable. Firms striving to achieve three-sigma performance use a critical value for the ratio of 1.0. A firm targeting four-sigma performance will use 1.33 (or 4/3), a firm targeting five-sigma performance will use 1.67 (or 5/3), and a firm striving for six-sigma performance will use 2.00 (or 6/3). Processes producing services or products with less than three-sigma performance will have C_{pk} values less than 1.0.

If a process passes the process capability index test, we can declare the process is capable. Suppose a firm desires its processes to produce at the level of four-sigma performance. If C_{pk} is greater than or equal to the critical value of 1.33, we can say the process is capable. If C_{pk} is less than the critical value, either the process average is too close to one of the tolerance limits and is generating defective output, or the process variability is too large. To find out whether the variability is the culprit, we need another test.

Process Capability Ratio If a process fails the process capability *index* test, we need a quick test to see if the process variability is causing the problem. If a process is *capable*, it has a process distribution whose extreme values fall within the upper and lower specifications for a service or

$$C_p = \frac{\text{Upper specification} - \text{Lower specification}}{6\sigma}$$

CHAPTER 5: MANAGING PROCESS CONSTRAINTS

Constraint: any factor that limits the performance of a system and restricts its output.

Bottleneck: a capacity restraint resource whose available capacity limits the organization's ability to meet the product volume, product mix, or demand fluctuation required by the marketplace.

Three kinds of constraints:

1. Physical
2. Market
3. Managerial

Theory of constraints: A systematic management approach that focuses on actively managing those constraints that impede a firm's progress toward its goal.

Key principles:

1. The focus should be on balancing flow, not on balancing capacity.
2. Maximizing the output and efficiency of every resource may not maximize the throughput of the entire system.
3. An hour lost at a bottleneck or constrained resource is an hour lost for the whole system.
4. Inventory is needed only in front of bottlenecks and in front of assembly and shipping points.
5. Work should be released into the system only as frequently as needed by the bottlenecks.
6. Activating a non-bottleneck resource is not the same as utilizing a bottleneck resource.
7. Every capital investment must be viewed from the perspective of the global impact on overall throughput, inventory, and operating expense.

Implementation steps:

1. Identify the System Bottleneck(s)
2. Exploit the Bottleneck(s)
3. Subordinate All Other Decisions to Step 2
4. Elevate the Bottleneck(s)
5. Do Not Let Inertia Set In

Managing bottlenecks in service processes:

- **Throughput:** Total elapsed time from the start to the finish of a job or a customer being processed at one or more work centers.
- **Bottlenecks either:**
 - Have the highest total time per unit processed
 - Or have the highest average utilization and total workload
- If it is difficult to identify bottlenecks, set low utilization rates to absorb unexpected surges in demand

The traditional vs. the bottleneck method application to product mix decisions:

- **Traditional method:** produce the products with the highest contribution margins or unit sales.
- **Bottleneck method:** the firm's profitability depends more upon the contribution margin generated at the bottleneck than by the contribution margin of each individual product produced.

Line balancing: the assignment of work to stations in a line process to achieve the desired output rate with the smallest number of workstations. Its goal is to match the output rate to the staffing or production plan.

Key Equations

Managing Constraints in Line Processes

1. Cycle time: $c = \frac{1}{r}$
2. Theoretical minimum number of workstations: $TM = \frac{\sum t}{c}$
3. Idle time: $nc - \sum t$
4. Efficiency(%): $\frac{\sum t}{nc}(100)$
5. Balance delay (%): $100 - \text{Efficiency}$

CHAPTER 6: DESIGNING LEAN SYSTEMS

Lean systems: operations systems that maximize the value added by each of a company's activities by removing waste and delays from them.

JIT philosophy: The belief that waste can be eliminated by cutting unnecessary capacity or inventory and removing non-value-added activities in operations.

The 8 types of MUDA:

1. Overproduction
2. Inappropriate Processing
3. Waiting
4. Transportation
5. Motion
6. Inventory
7. Defects
8. Underutilization of Employees

MUDA == waste.

KAIZEN == continuous improvement.

JIDOKA == automatically stopping the process when something is wrong and then fixing the problems on the line as they occur.

POKE-YOKE == mistake proofing methods aimed at designing fail-safe systems that minimize human error.

ANDON == a system that gives machines and machine operators the ability to signal the occurrence of any abnormal condition.

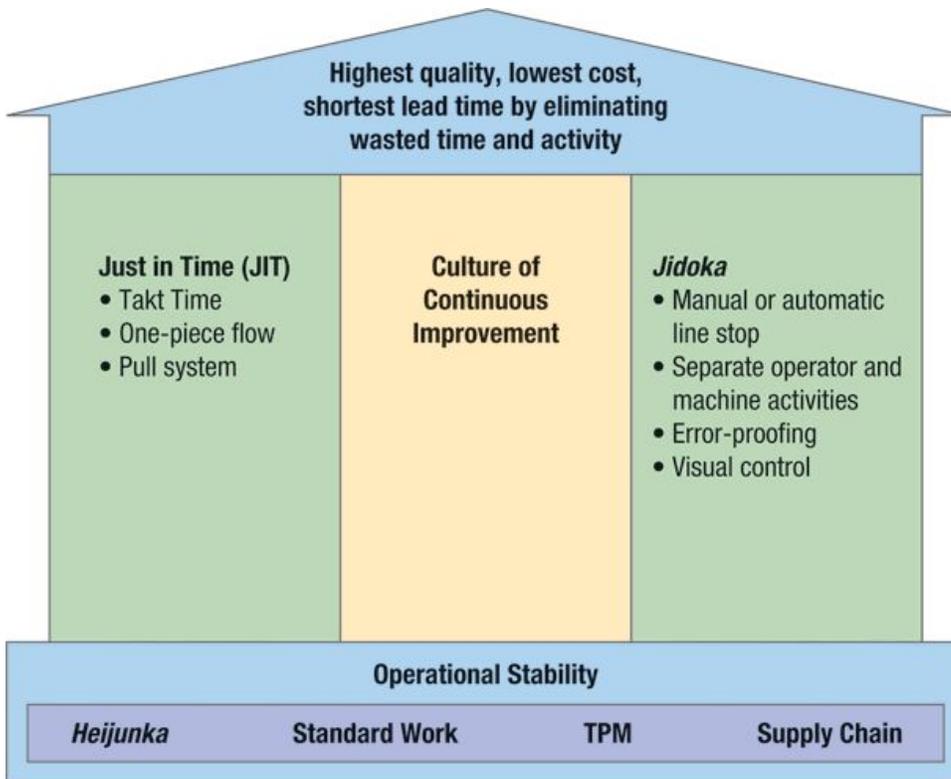
HEIJUNKA == the levelling of production load by both volume and product mix.

KANBAN == card.

Strategic characteristics of lean systems:

1. Supply chain considerations in lean systems
 - 1.1. Close supplier ties
 - 1.1.1. JIT 2 system
 - 1.2. Small lot sizes
 - 1.2.1. Advantages
 - 1.2.1.1. Reduce inventory
 - 1.2.1.2. Reduce discovery time of defects
 - 1.2.1.3. Uniform workload
 - 1.2.2. Disadvantages
 - 1.2.2.1. Increased setup time
2. Process considerations in lean systems
 - 2.1. Pull method of work flow
 - 2.2. Quality at the source
 - 2.2.1. JIDOKA
 - 2.2.2. POKA-YOKE
 - 2.2.3. ANDON
 - 2.3. Uniform workstation loads
 - 2.3.1. For service processes
 - 2.3.1.1. Reservation systems
 - 2.3.1.2. Differential pricing
 - 2.3.2. For manufacturing processes
 - 2.3.2.1. Same type and number of units each day
 - 2.3.3. HEIJUNKA
 - 2.3.4. Mixed-model assembly
 - 2.4. Standardized components and work methods
 - 2.5. Flexible workforce
 - 2.6. Automation
 - 2.7. 5S
 - 2.7.1. Sort
 - 2.7.2. Straighten
 - 2.7.3. Shine
 - 2.7.4. Standardize
 - 2.7.5. Sustain
 - 2.8. Total preventive maintenance

Toyota production system

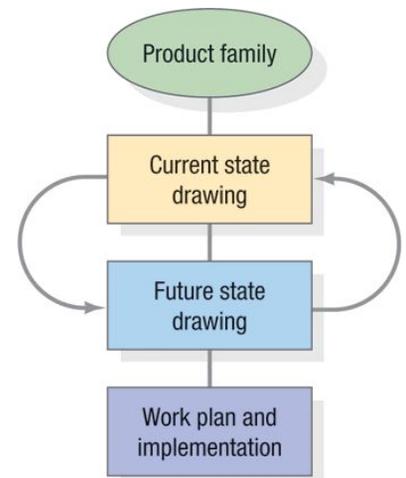


Designing lean system layouts:

1. One worker, multiple machines
2. Group technology

Value stream mapping:

- A widely used qualitative lean tool aimed at eliminating waste or MUDA.
- Unlike the theory of constraints which accepts the existing system bottlenecks and then strives to maximize the throughput given that set of constraints.
- It tries to understand through current and future state maps how existing processes can be altered to eliminate bottlenecks and other wasteful activities.



Determining the number of containers in KANBAN:

Work in Process (WIP) =
(average demand rate) × (average time a container spends
in the manufacturing process) + safety stock

$$WIP = kc$$

where

$$kc = \bar{d} (\bar{w} + \bar{p}) (1 + \alpha)$$

$$k = \frac{\bar{d} (\bar{w} + \bar{p}) (1 + \alpha)}{c}$$

k = number of containers

\bar{d} = expected daily demand for the part

\bar{w} = average waiting time

\bar{p} = average processing time

c = number of units in each container

α = policy variable

Organizational benefits and implementation issues of lean systems:

1. Organizational considerations
 - 1.1. The human costs of lean systems
 - 1.2. Cooperation and trust
 - 1.3. Reward systems and labor classifications
2. Process considerations
3. Inventory and scheduling
 - 3.1. Schedule stability
 - 3.2. Setups
 - 3.3. Purchasing and logistics

CHAPTER 7: MANAGING EFFECTIVE PROJECTS

Project: an interrelated set of activities with a definite starting and ending point, which results in a unique outcome for a specific allocation of resources.

Project management: a systemized, phased approach to defining, organizing, planning, monitoring, and controlling projects.

Program: an interdependent set of projects that have a common strategic purpose.

Defining and organizing projects:

1. Defining the scope and objectives of a project
2. Selecting the project manager and team
 - 2.1. Project manager
 - 2.1.1. Facilitator
 - 2.1.2. Communicator
 - 2.1.3. Decision maker
 - 2.2. Project team
 - 2.2.1. Technical competence
 - 2.2.2. Sensitivity
 - 2.2.3. Dedication
3. Recognizing organization structure
 - 3.1. Functional
 - 3.2. Pure project
 - 3.3. Matrix

Network diagram: a network planning method designed to depict the relationships between activities, that consist of nodes (circles) and arcs (arrows).

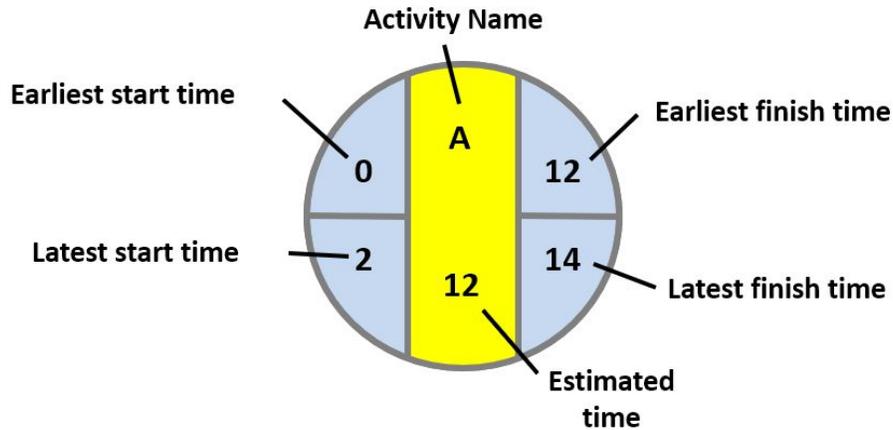
Estimating activity times:

- Statistical methods
- Learning curve models
- Managerial opinions

Developing the project schedule:

Path: the sequence of activities between a project's start and finish.

Critical path: the sequence of activities between a project's start and finish that takes the longest time to complete.



- **ES** = latest previous activity EF
- **LS** = $LF - t$
- **EF** = $ES + t$
- **LF** = earliest next activity LS
- **S** (i.e., slack)
 - = $LS - ES$
 - = $LF - EF$

Total project cost:

1. Direct
2. Indirect
3. Penalty

Project crashing: shortening (or expediting) some activities within a project to reduce overall project completion time and total project costs. The following need to be known:

- NT
- NC
- CT
- CC

$$\text{Cost to crash per period} = (CC - NC) / (NT - CT)$$

Minimum cost schedule: a schedule determined by starting with the normal time (NC) schedule and crashing activities along the critical path, in such a way that the costs of crashing do not exceed the savings in indirect and penalty costs.

1. Determine the project's critical path(s).
2. Find the activity or activities on the critical path(s) with the lowest cost of crashing per week.
3. Reduce the time for this activity until...
 - 3.1. It cannot be further reduced or
 - 3.2. Another path becomes critical, or
 - 3.3. The increase in direct costs exceeds the indirect and penalty cost savings that result from shortening the project.
4. Repeat this procedure until the increase in direct costs is larger than the savings generated by shortening the project.

Assessing and analyzing risks:

1. Risk-management plans
 - 1.1. Strategic Fit
 - 1.2. Service/Product Attributes
 - 1.3. Project Team Capability
 - 1.4. Operations
2. Statistical Analysis
 - 2.1. Optimistic time (a)
 - 2.2. Most likely time (m)
 - 2.3. Pessimistic time (b)

Project close out: an activity that includes writing final reports, completing remaining deliverables, and compiling the team's recommendations for improving the project process.

Key Equations

Developing the Project Schedule

1. Start and finish times:

$$\begin{aligned} t &= \text{estimated time duration of the activity} \\ ES &= \text{latest of the EF times of all activities immediately preceding activity} \\ EF &= ES + t \\ LF &= \text{earliest of the LS times of all activities immediately following activity} \\ LS &= LF - t \end{aligned}$$

2. Activity slack:

$$S = LS - ES \text{ or } S = LF - EF$$

Analyzing Cost-Time Trade-offs

3. Project costs:

$$\begin{aligned} \text{Crash cost per period} &= \frac{\text{Crash cost} - \text{Normal cost}}{\text{Normal time} - \text{Crash time}} \\ &= \frac{CC - NC}{NT - CT} \end{aligned}$$

Assessing and Analyzing Risks

4. Activity time statistics:

t_e = mean of an activity's beta distribution

$$t_e = \frac{a + 4m + b}{6}$$

σ^2 = variance of the activity time

$$\sigma^2 = \left(\frac{b - a}{6} \right)^2$$

5. z-transformation formula:

$$z = \frac{T - T_E}{\sigma_P}$$

where

T = due date for the project

T_E = \sum (expected activity times on the critical path)

= mean of normal distribution of critical path time

σ_P = standard deviation of critical path time distribution

CHAPTER 8: FORECASTING DEMAND

Forecast: a prediction of future events used for planning purposes.

Demand patterns:

1. Horizontal
2. Trend
3. Seasonal
4. Cyclical
5. Random

Demand management options:

1. Complementary Products
2. Promotional Pricing
3. Prescheduled Appointments
4. Reservations
5. Revenue Management
6. Backlogs
7. Backorders and Stockouts

Key decisions on making forecasts:

1. Deciding what to forecast
 - 1.1. Level of aggregation
 - 1.2. Unit of measurement
2. Choosing the type of forecasting technique
 - 2.1. Judgment methods
 - 2.1.1. Salesforce estimates
 - 2.1.2. Executive opinion
 - 2.1.3. Market research
 - 2.1.4. Delphi method
 - 2.2. Causal methods
 - 2.2.1. Linear regression
 - 2.3. Time-series analysis
 - 2.3.1. Naïve forecast
 - 2.3.2. Horizontal patterns
 - 2.3.2.1. Simple moving average
 - 2.3.2.2. Weighted moving average
 - 2.3.2.3. Exponential smoothing
 - 2.3.3. Trend patterns (using regression)
 - 2.3.4. Seasonal patterns (using seasonal factors)

Forecast error:

		Forecast	Error	Error	Error ²	Pct Error
Past period 1	39	41	-2	2	4	5.128%
Past period 2	37	43	-6	6	36	16.216%
Past period 3	55	45	10	10	100	18.182%
Past period 4	40	50	-10	10	100	25%
Past period 5	59	51	8	8	64	13.559%
Past period 6	63	56	7	7	49	11.111%
Past period 7	41	61	-20	20	400	48.78%
Past period 8	57	60	-3	3	9	5.263%
Past period 9	56	62	-6	6	36	10.714%
Past period 10	54	63	-9	9	81	16.667%
TOTALS	501		-31	81	879	170.621%
AVERAGE	50.1		-3.1	8.1	87.9	17.062%
			(Bias)	(MAD)	(MSE)	(MAPE)
				Std dev	29.648	

Criteria for selecting time-series methods:

1. Using statistical criteria
 - 1.1. If stable
 - 1.1.1. Low alpha
 - 1.1.2. Large n
 - 1.2. If dynamic
 - 1.2.1. High alpha
 - 1.2.2. Small n
2. Using a holdout sample
3. Using a tracking signal

Forecasting principles:

1. Better processes yield better forecasts
2. Demand forecasting is being done in virtually every company, either formally or informally. The challenge is to do it well—better than the competition
3. Better forecasts result in better customer service and lower costs, as well as better relationships with suppliers and customers
4. The forecast can and must make sense based on the big picture, economic outlook, market share, and so on
5. The best way to improve forecast accuracy is to focus on reducing forecast error
6. Bias is the worst kind of forecast error; strive for zero bias
7. Whenever possible, forecast at more aggregate levels. Forecast in detail only where necessary
8. Far more can be gained by people collaborating and communicating well than by using the most advanced forecasting technique or model

Key Equations

Forecast Error

1. Forecast error measures:

$$E_t = D_t - F_t$$

$$\text{CFE} = \sum E_t$$

$$\bar{E} = \frac{\text{CFE}}{n}$$

$$\text{MSE} = \frac{\sum E_t^2}{n}$$

$$\sigma = \sqrt{\frac{\sum (E_t - \bar{E})^2}{n - 1}}$$

$$\text{MAD} = \frac{\sum |E_t|}{n}$$

$$\text{MAPE} = \frac{(\sum |E_t| / D_t)(100\%)}{n}$$

Causal Methods: Linear Regression

2. Linear regression:

$$Y = a + bX$$

Time-Series Methods

3. Naïve forecasting:

$$\text{Forecast} = D_t$$

4. Simple moving average:

$$F_{t+1} = \frac{D_t + D_{t-1} + D_{t-2} + \dots + D_{t-n+1}}{n}$$

5. Weighted moving average:

$$F_{t+1} = \text{Weight}_1(D_t) + \text{Weight}_2(D_{t-1}) + \text{Weight}_3(D_{t-2}) + \dots + \text{Weight}_n(D_{t-n+1})$$

6. Exponential smoothing:

$$F_{t+1} = \alpha D_t + (1 - \alpha)F_t$$

7. Trend Projection using Regression

$$F_t = a + bt$$

8. Tracking signal:

$$\frac{\text{CFE}}{\text{MAD}} \text{ or } \frac{\text{CFE}}{\text{MAD}_t}$$

9. Exponentially smoothed error:

$$\text{MAD}_t = \alpha |E_t| + (1 - \alpha)\text{MAD}_{t-1}$$